

**ASPECTS OF ELECTROWEAK PHYSICS FOR A COMPOSITE HIGGS BOSON;
APPLICATION TO THE Z GAUGE BOSON DECAYING INTO
TWO LEPTONS AND TWO PSEUDOSCALAR MESONS**

B. Machet ^{1 2}

*Laboratoire de Physique Théorique et Hautes Énergies ³
Universités Pierre et Marie Curie (Paris 6) et Denis Diderot (Paris 7)
Unité associée au CNRS UMR 7589*

Abstract: I study phenomenological aspects of the $SU(2)_L \times U(1)$ electroweak physics of a Higgs boson transforming like a quark-antiquark pair. A correspondence is established between its flavour content, the hierarchy of quark condensates, and the leptonic decay constants f of pseudoscalar mesons; the Higgs-mesons couplings coming from the symmetry-breaking scalar potential can then be expressed in terms of the f 's, of the Cabibbo-Kobayashi-Maskawa mixing angles, and of the mass of the Higgs boson. Application is made to the decays of a Z boson into two leptons and two charged pseudoscalar mesons, more specially $e^+e^-B^+B^-$, $e^+e^-D_s^+D_s^-$ and $e^+e^-K^\pm\pi^\mp$; the last channel, involving new types of flavour changing neutral currents in the scalar sector, is characteristic of this approach. Unlike in the standard model for quarks, the detection of two outgoing charged B mesons is hopeless, like, unfortunately, that of $K^\pm\pi^\mp$; the channel $D_s^+D_s^-$ could have been missed in present experiments for a Higgs mass lower than 65 GeV because of insufficient statistics. For higher masses this Higgs becomes undetectable from such channels. This shows that too drastic conclusions should not be drawn from an eventual negative outcome from searches for the standard Higgs boson, that simple variants of it are more elusive, may have escaped detection, and could still escape in the near future.

PACS: 11.15.Ex, 12.60.Fr, 12.60.Rc, 13.38.Dg, 13.20.-v, 14.80.Cp



¹Member of 'Centre National de la Recherche Scientifique'

²E-mail: machet@lpthe.jussieu.fr

³LP THE tour 16 / 1^{er} étage, Université P. et M. Curie, BP 126, 4 place Jussieu, F-75252 PARIS CEDEX 05 (France)

1 Introduction

The experimental search [1] for the Higgs [2] boson is mainly concerned nowadays by that of its strictly standard (unique) avatar [3] or of its multiple supersymmetric [4] reincarnations. The present and next generations of accelerators should be able to give us precious information about this cornerstone of particle physics.

If a positive outcome would undoubtedly be one of the greatest achievements of the century and provide a fair reward for the simplest and most elegant ideas, a negative one should not be considered as a failure and be interpreted as the absence of the Higgs but rather as an encouragement to refine our ideas, to look for alternatives, and in particular to investigate whether one could have missed a Higgs particle with couplings smaller than predicted in the basic models.

This is the goal of this work, which gives an example of a nearly standard Higgs boson, still considered to be unique, which could have escaped detection.

I outpass here the present negative feelings attached to technicolour models [5] (mainly due to the problem of flavour changing neutral currents) and revive a composite Higgs boson; however, if it does transform like a quark-antiquark pair, it nevertheless appears, unlike the quarks, as a fundamental field in the Lagrangian, together with the observed pseudoscalar (and scalar) mesons, along the lines of [6]. It is a neat way of including both chiral and electroweak properties of asymptotic states. As the proposed model respects in particular the standard electroweak transformations of quarks, the customary problems associated with flavour changing neutral currents do not arise (we shall see that a new type of them occurs but only in the Higgs sector, and beyond our present detection capabilities).

The only concern being here the electroweak physics of scalar (Higgs) and pseudoscalar $J = 0$ states, Quantum Chromodynamics (QCD) [7] and strong interactions are deliberately left aside, except in that they determine the asymptotic mesons and their normalization; interactions among particles in the final state are not considered. The quark picture is often invoked, but only to make the reader more comfortable and to provide a link with customary considerations: the quarks themselves, which are not asymptotic fields, do not appear in the Lagrangian. The breakdown of the electroweak symmetry, tantamount to a condensation of quark-antiquark pairs, is treated on a purely phenomenological ground (much alike in the standard electroweak model) without reference to a deeper underlying mechanism.

The Higgs boson being searched for through its decays, determining its coupling to *observed* particles is essential. Yukawa couplings to quarks are now absent, but the Higgs couples through a “mexican hat” potential to the three goldstones of the broken electroweak symmetry, which are, as is shown by the study of their leptonic decays, linear combinations of pseudoscalar mesons. There exist $N^2/4$ ($N/2$ is the number of generations) real electroweak quadruplets of scalar and pseudoscalar (composite) fields isomorphic to the complex isoscalar doublet of the Glashow-Salam-Weinberg model [3]. Accordingly, the flavour orientation of the Higgs boson is determined by a rotation in this $N^2/4$ dimensional space. It determines, too, the flavour content of the three (pseudoscalar) Goldstone bosons which are the three partners of the Higgs in the same quadruplet, and the hierarchy between the leptonic decay constants f of the various pseudoscalar mesons. This makes possible the determination of the couplings of the Higgs to pseudoscalar mesons as a function of the f 's, of Cabibbo-Kobayashi-Maskawa (CKM) [8] mixing angles and of the mass of the Higgs itself.

Its couplings to gauge fields keep as in the standard model.

I also show how, in an interpretation in terms of quarks, the orientation of the Higgs boson determines the hierarchy of the different quark-antiquark vacuum expectation values.

The paper is organized as follows:

- in section 2, I recall the theoretical basis of the model;
- in section 3, I study the orientation of the Higgs boson in flavour space and its consequences;
- in section 4, I specialize to the three decays $Z \rightarrow e^+e^-B^+B^-$, $Z \rightarrow e^+e^-D_s^+D_s^-$ and $Z \rightarrow e^+e^-K^\pm\pi^\mp$;
- finally, appendix A gives technical details concerning the basis of electroweak and flavour eigenstates, and appendix B provides a link between the normalization of the asymptotic mesons and the Gell-Mann-Oakes-Renner relation [9] in QCD.

2 Electroweak interactions of quark-antiquark composite fields

2.1 Electroweak and “flavour” eigenstates

Though we only deal here with electroweak physics, the paper rests on the fact that the “asymptotic” states for $J = 0$ mesons can be interpreted as “flavour” eigenstates, determined by strong interactions¹.

This can be put in correspondence with the existence of two different mass scales, hence two different characteristic times:

- the electroweak mass scale $M_W \approx 80$ GeV, with an associated time scale $\tau_{EW} = 1/M_W$;
- the mass scale associated with strong interactions, with an order of magnitude of the masses of the mesons and resonances exchanged in nuclear interactions, that is a few hundred MeV; the associated time scale τ_S is much larger than τ_{EW} ; thus, if an electroweak eigenstate is produced (by electroweak interactions) at time t it can only be detected as such between t and $t + \tau_{EW}$; after this interval, and before it eventually decays into final states which can be non-hadronic, one only detects its flavour components. The meaning of “asymptotic”, which has to be adapted to the type of problem that is being analyzed, is consequently here “for time scales larger than τ_{EW} ”.

While the customary procedure is to try to incorporate strong interactions between asymptotic electroweak eigenstates to build up observed particles, which of course, like when introducing gluonic corrections in QCD, faces non-perturbative problems, I will rather here consider as perturbative and small the electroweak interactions between flavour asymptotic states which are determined by strong interactions. The only additional (non-perturbative) effect of strong interactions that I will introduce is the normalization of asymptotic mesons which is determined from their leptonic decays and is shown to be in agreement with the “Partially Conserved Current Hypothesis” (PCAC) and with the Gell-Mann-Oakes-Renner relation in QCD (see subsections 2.2.1, 2.2.4, appendix B)².

2.2 Theoretical framework

The general framework has been set in [6]. For the sake of understandability and for this paper to be self-contained, I briefly recall here the main useful steps, in a somewhat less formal approach more usable for phenomenological purposes.

Quarks are considered to be mathematical objects [10] which are determined by their quantum numbers and by their transformations by the different groups of symmetry that act upon them; we are mainly concerned here with the chiral group $U(N)_L \times U(N)_R$ where N is the number of “flavours”, and with the electroweak group $SU(2)_L \times U(1)$: they form an N -vector Ψ

$$\Psi = \begin{pmatrix} u \\ c \\ \vdots \\ d \\ s \\ \vdots \end{pmatrix} \quad (1)$$

in the fundamental representation of the diagonal subgroup of the chiral group, and their electroweak transformations, to which we stick to, are the usual ones of the Glashow-Salam-Weinberg model [3].

Any $SU(2)_L \times U(1)$ group can be considered, for N even, as a subgroup of $U(N)_L \times U(N)_R$; that it be the electroweak group, *i.e.* that it act on quarks in the standard way determines its embedding in the chiral group. It is easy to check that the $SU(2)$ group the three generators of which are the three $N \times N$ matrices (they are written in terms of four $N/2 \times N/2$ sub-blocs with \mathbb{I} the unit matrix and \mathbb{K} the CKM mixing matrix)

$$\mathbb{T}_L^3 = \frac{1}{2} \left(\begin{array}{c|c} \mathbb{I} & 0 \\ \hline 0 & -\mathbb{I} \end{array} \right), \quad \mathbb{T}_L^+ = \left(\begin{array}{c|c} 0 & \mathbb{K} \\ \hline 0 & 0 \end{array} \right), \quad \mathbb{T}_L^- = \left(\begin{array}{c|c} 0 & 0 \\ \hline \mathbb{K}^\dagger & 0 \end{array} \right), \quad (2)$$

¹Strong interactions are considered to be independent of “flavour”, and so both have common eigenstates.

² The question can be raised whether the Higgs can appear as an asymptotic state or is projected at “large” times on flavour (scalar) eigenstates; its eventual detectability as an asymptotic particle can depend on it.

acting trivially on the left-handed projection $\Psi_L = [(1 - \gamma_5)/2]\Psi$ of Ψ can be identified with the standard electroweak $SU(2)_L$; the $U(1)$ associated to the weak hypercharge \mathbb{Y} is determined through the Gell-Mann-Nishijima relation [11]

$$(\mathbb{Y}_L, \mathbb{Y}_R) = (\mathbb{Q}_L, \mathbb{Q}_R) - (\mathbb{T}_L^3, 0), \quad (3)$$

and from the trivial form for the (diagonal) charge operator \mathbb{Q}

$$\mathbb{Q}_L = \mathbb{Q}_R = \mathbb{Q} = \left(\begin{array}{c|c} 2/3 & 0 \\ \hline 0 & -1/3 \end{array} \right). \quad (4)$$

The $2N^2$ composite of the form $\bar{q}q$ or $\bar{q}\gamma_5 q$ can be cast into $N^2/2$ quadruplets which are stable by the electroweak group; their flavour structure is materialized by $N \times N$ matrices \mathbb{M} and the quadruplets can generically be written

$$\begin{aligned} \Phi(\mathbb{D}) &= (\mathbb{M}^0, \mathbb{M}^3, \mathbb{M}^+, \mathbb{M}^-)(\mathbb{D}) \\ &= \left[\frac{1}{\sqrt{2}} \left(\begin{array}{c|c} \mathbb{D} & 0 \\ \hline 0 & \mathbb{K}^\dagger \mathbb{D} \mathbb{K} \end{array} \right), \frac{i}{\sqrt{2}} \left(\begin{array}{c|c} \mathbb{D} & 0 \\ \hline 0 & -\mathbb{K}^\dagger \mathbb{D} \mathbb{K} \end{array} \right), i \left(\begin{array}{c|c} 0 & \mathbb{D} \mathbb{K} \\ \hline 0 & 0 \end{array} \right), i \left(\begin{array}{c|c} 0 & 0 \\ \hline \mathbb{K}^\dagger \mathbb{D} & 0 \end{array} \right) \right], \end{aligned} \quad (5)$$

where \mathbb{D} is a real $N/2 \times N/2$ matrix (see subsection 2.2.2 and Appendix A). One may furthermore consider quadruplets the entries of which have a definite parity (the \mathbb{S} 's below stand for scalars and the \mathbb{P} 's for pseudoscalars)

$$\varphi = (\mathbb{S}^0, \vec{\mathbb{P}}), \quad (6)$$

and

$$\chi = (\mathbb{P}^0, \vec{\mathbb{S}}). \quad (7)$$

The φ 's and the χ 's transform alike by the gauge group, according to (i and j are $SU(2)$ indices)

$$\begin{aligned} \mathbb{T}_L^i \cdot \mathbb{M}^j &= -\frac{i}{2} (\epsilon_{ijk} \mathbb{M}^k + \delta_{ij} \mathbb{M}^0), \\ \mathbb{T}_L^i \cdot \mathbb{M}^0 &= \frac{i}{2} \mathbb{M}^i. \end{aligned} \quad (8)$$

The link between the matrices \mathbb{M} and diquark operators is straightforwardly established by sandwiching the latter between $\bar{\Psi}$ and Ψ and inserting a γ_5 when needed by parity.

The link between diquark operators, of dimension $[mass]^3$, and the scalar (pseudoscalar) fields (electroweak eigenstates) of dimension $[mass]$ occurring in the Lagrangian is achieved by introducing an appropriate normalization as follows.

Let $H = h + \langle H \rangle$ be the Higgs boson such that

$$\langle H \rangle = \frac{v}{\sqrt{2}} \quad (9)$$

breaks the electroweak $SU(2)_L \times U(1)$ into its electromagnetic $U(1)_{em}$ subgroup. Its flavour content is represented by an $N \times N$ matrix \mathbb{H} and the associated diquark operator is $\bar{\Psi} \mathbb{H} \Psi$. Eq. (9) is trivially satisfied by

$$H = \frac{\langle H \rangle}{\langle \bar{\Psi} \mathbb{H} \Psi \rangle} \bar{\Psi} \mathbb{H} \Psi \quad (10)$$

from which one can choose for all fields ϕ with dimension $[mass]$ associated with the $N \times N$ matrix \mathbb{M} the normalization

$$\phi_{\mathbb{M}} = (i) \frac{\langle H \rangle}{\langle \bar{\Psi} \mathbb{H} \Psi \rangle} \bar{\Psi} (\gamma_5) \mathbb{M} \Psi. \quad (11)$$

2.2.1 Flavour eigenstates

Because of the CKM rotation, the electroweak eigenstates φ and χ defined in (6,7) are not flavour eigenstates but linear combinations of them.

The flavour or “strong” eigenstates are the ones associated with \mathbb{M} matrices which have only one nonvanishing entry equal to 1. Let \mathbb{M}_{ab} such a matrix with one single non-vanishing entry at the crossing of the a -th line and b -th column. The associated flavour eigenstate, that we call Π_{ab} is, according to (11), and in the case of a pseudoscalar

$$\Pi_{ab}(\mathbb{M}_{ab}) = i \frac{\langle H \rangle}{\langle \bar{\Psi} \mathbb{H} \Psi \rangle} \bar{\Psi} \gamma_5 \mathbb{M}_{ab} \Psi. \quad (12)$$

The Π_{ab} ’s are related to asymptotic states, *i.e.* observed mesons \mathcal{P}_{ab} , by a scaling factor \mathbf{b} (see subsection 2.2.4).

2.2.2 Quadratic invariants and electroweak mass scales

To every quadruplet $(\mathbb{M}^0, \vec{\mathbb{M}})$ is associated a quadratic invariant:

$$\mathcal{I} = (\mathbb{M}^0, \vec{\mathbb{M}}) \otimes (\mathbb{M}^0, \vec{\mathbb{M}}) = \mathbb{M}^0 \otimes \mathbb{M}^0 + \vec{\mathbb{M}} \otimes \vec{\mathbb{M}}; \quad (13)$$

the “ \otimes ” product is a tensor product (not the usual multiplication of matrices) and means the product of fields as functions of space-time; $\vec{\mathbb{M}} \otimes \vec{\mathbb{M}}$ stands for $\sum_{i=1,2,3} \mathbb{M}^i \otimes \mathbb{M}^i$.

For the relevant cases $N = 2, 4, 6$, there exists a set of \mathbb{D} real matrices (see appendix A) such that the algebraic sum of invariants specified below, extended over all representations defined by (6,7,5)

$$\frac{1}{2} \left(\left(\sum_{\text{symmetric } \mathbb{D}'s} - \sum_{\text{antisym } \mathbb{D}'s} \right) \left((\mathbb{S}^0, \vec{\mathbb{P}})(\mathbb{D}) \otimes (\mathbb{S}^0, \vec{\mathbb{P}})(\mathbb{D}) - (\mathbb{P}^0, \vec{\mathbb{S}})(\mathbb{D}) \otimes (\mathbb{P}^0, \vec{\mathbb{S}})(\mathbb{D}) \right) \right) \quad (14)$$

is diagonal both in the electroweak basis and in the basis of flavour eigenstates. With the coefficient $(1/2)$ chosen in (14) in the electroweak basis, the normalization in the basis of flavour eigenstates is $(+1)$, with all signs positive.

From the property stated above, to each quadruplet can be associated an arbitrary electroweak mass scale and, for such a choice of \mathbb{D} matrices, the degeneracies of electroweak and flavour eigenstates coincide.

2.2.3 The electroweak Lagrangian

The scalar (pseudoscalar) electroweak fields that build up the Lagrangian are taken to be the ones associated with the set of matrices \mathbb{D} diagonalizing the invariant (14) in both the electroweak and the flavour basis, and the combination used for the kinetic terms is the one of (14).

The kinetic terms for the leptons and the gauge fields are the standard ones.

No Yukawa coupling to quarks is present since they are not fields of the Lagrangian, and masses are given in a gauge invariant way to the mesons themselves³.

A “mexican hat” potential is phenomenologically introduced to trigger the spontaneous symmetry breaking of the electroweak symmetry.

³The hadronic sector being anomaly-free, a purely vectorial theory in the leptonic sector is then favoured; the approach proposed in [12] is an example of that, where leptons are given masses without introducing Yukawa couplings and where the standard model appears as an effective theory when one neutrino helicity decouples by becoming infinitely massive.

2.2.4 Normalizing the fields

By convention, we take all electroweak eigenstates ϕ occurring in the Lagrangian and defined by (11) to be normalized to “1”:

$$\int \frac{d^4 p}{(2\pi)^3} |\phi(p)\rangle \langle \phi(p)| \theta(p_0) \delta(p^2 - m_\phi^2) = 1, \quad (15)$$

$$\langle \phi(p') | \phi(p) \rangle = 2p_0 (2\pi)^3 \delta^3(p - p'), \quad (16)$$

and, for fields corresponding to two different \mathbb{D} matrices

$$\langle \phi(\mathbb{D}_\alpha) | \phi(\mathbb{D}_\beta) \rangle = 0, \quad \alpha \neq \beta. \quad (17)$$

The phase-space measure for ϕ is

$$d\mu_{\phi(p)} = \frac{d^4 p}{(2\pi)^3} \theta(p_0) \delta(p^2 - m_\phi^2). \quad (18)$$

The flavour eigenstates Π_{ab} defined in (12) are then normalized to $(1/2)$

$$\langle \Pi(p') | \Pi(p) \rangle = \left(\frac{1}{2}\right) 2p_0 (2\pi)^3 \delta^3(p - p'); \quad (19)$$

the difference of normalization between the ϕ 's and the Π 's comes from the identity

$$\sum_{\alpha} \mathbb{D}_{\alpha} \otimes \mathbb{D}_{\alpha} = 2 \sum_{ab} \mathbb{D}_{ab} \otimes \mathbb{D}_{ab} \quad (20)$$

where the \mathbb{D}_{α} 's are the ones exhibited in appendix A and the \mathbb{D}_{ab} 's are $N/2 \times N/2$ matrices the only non-vanishing entry of which is “1” at the crossing of the a -th line and b -th column; this property also reflects into the mismatch in the normalizations when expressing the quadratic invariant (14) in terms of electroweak or flavour eigenstates, as stated in subsection 2.2.2.

The last point concerns the normalization of the asymptotic (observed) mesons \mathcal{P}_{ab} ($\mathcal{P}_{ud}^+ = \pi^+$, $\mathcal{P}_{su}^- = K^- \dots$) which have the same flavour structure as the Π_{ab} 's: one introduces the scaling factor \mathfrak{b} such that

$$\Pi = \mathfrak{b} \mathcal{P}. \quad (21)$$

For the \mathcal{P} 's one has

$$\langle \mathcal{P}(p') | \mathcal{P}(p) \rangle = \frac{1}{2\mathfrak{b}^2} 2p_0 (2\pi)^3 \delta^3(p - p') \quad (22)$$

and

$$d\mu_{\mathcal{P}(p)} = 2\mathfrak{b}^2 \frac{d^4 p}{(2\pi)^3} \theta(p_0) \delta(p^2 - m_{\mathcal{P}}^2). \quad (23)$$

The scaling factor \mathfrak{b} is determined from the leptonic decays of the pseudoscalar mesons and is ⁴

$$\mathfrak{b} = \frac{\langle H \rangle}{2f_0} \quad (24)$$

where f_0 is the generic leptonic decay constant.

As a consequence, one has for example (for $N = 4$)

$$P^+(\mathbb{D}_1) = \mathfrak{b} (\cos \theta_c (\pi^+ + D_s^+) + \sin \theta_c (K^+ - D^+)), \quad (25)$$

where θ_c is the Cabibbo angle.

I show in Appendix B how this scaling is consistent, through PCAC, with the Gell-Mann-Oakes-Renner relation [9] in QCD.

The scaling factor \mathfrak{b} , that we introduce in a non-perturbative way, and which ensures the consistency with other approaches and with experimental observations, can be put in parallel with the \sqrt{Z} 's normalizing asymptotic fields in the S -matrix theory; our hypothesis lies here in that, once admitted that strong interactions determine asymptotic states, their only other effect can be phenomenologically parameterized by \mathfrak{b} ⁵.

⁴In previous works [13] the asymptotic mesons were normalized to “1”, which led to a scaling factor $a = 1/\mathfrak{b}$. Though the physical results turn out, as expected to be the same, the procedure used here is more systematic and makes the links more conspicuous with the traditional picture of QCD.

⁵We in particular do not consider here the phase-shifts introduced by strong interactions among final states.

3 Electroweak symmetry breaking and the Higgs boson

3.1 The flavour orientation of the Higgs boson

The real (H, \vec{G}) quadruplet (complex doublet) of the standard model, where H is the (scalar) Higgs boson and $\vec{G} = (G^+, G^3, G^-)$ are the three goldstones of the broken electroweak symmetry is isomorphic to any of the $N^2/4$ quadruplets φ of eq. (6) ⁶.

We thus face an arbitrariness in the flavour content of the Higgs boson, which also determines the composition of the three goldstones in terms of flavour eigenstates since they are its three pseudoscalar partners in the same quadruplet.

Identifying H with $\mathbb{S}^0(\mathbb{D}_1)$ as I did in previous works [13] is tantamount to taking the same value for all diagonal vacuum expectation values $\langle \bar{q}_a q_a \rangle$ (a is a flavour index). I loosen here this hypothesis and introduce a real orthogonal rotation matrix \mathcal{R} acting in the $N^2/4$ dimensional space of the φ 's.

For the sake of simplicity, I will perform the analysis in the case of two generations ($N = 4$). We can then restrict furthermore \mathcal{R} to be a 3×3 rotation matrix by postulating that $\langle \mathbb{S}^0(\mathbb{D}_4) \rangle = 0$; this is equivalent to saying that $\langle \bar{q}_a q_b - \bar{q}_b q_a \rangle = 0$, which is true if CP is an unbroken symmetry. So, the (H, \vec{G}) multiplet is now considered to be a linear combination of $\varphi_1 = \varphi(\mathbb{D}_1)$, $\varphi_2 = \varphi(\mathbb{D}_2)$ and $\varphi_3 = \varphi(\mathbb{D}_3)$ (see eq. (6)), and the 4×4 flavour matrix associated to G^+ reads

$$\mathbb{G}^+ = i \left(\begin{array}{c|cc} 0 & G_{ud}^+ & G_{us}^+ \\ \hline & G_{cd}^+ & G_{cs}^+ \\ \hline 0 & & 0 \end{array} \right). \quad (26)$$

Let the orthogonal matrix ⁷ \mathcal{R} such that

$$\begin{pmatrix} \tilde{\varphi}_1 \\ \tilde{\varphi}_2 \\ \tilde{\varphi}_3 \end{pmatrix} \equiv \begin{pmatrix} (\tilde{\mathbb{S}}_1, \vec{\tilde{\mathbb{P}}}_1) \\ (\tilde{\mathbb{S}}_2, \vec{\tilde{\mathbb{P}}}_2) \\ (\tilde{\mathbb{S}}_3, \vec{\tilde{\mathbb{P}}}_3) \end{pmatrix} = \mathcal{R} \begin{pmatrix} \varphi(\mathbb{D}_1) \\ \varphi(\mathbb{D}_2) \\ \varphi(\mathbb{D}_3) \end{pmatrix} \quad (28)$$

depend on three mixing angles $\theta_1, \theta_2, \theta_3$, the sines and cosines of which will be noted s_1, s_2, s_3 and c_1, c_2, c_3

$$\mathcal{R} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 & c_1 c_2 s_3 + s_2 c_3 \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 & c_1 s_2 s_3 - c_2 c_3 \end{pmatrix}. \quad (29)$$

We choose by convention ⁸

$$(H, \vec{G}) = \tilde{\varphi}_1 = (\tilde{\mathbb{S}}_1, \vec{\tilde{\mathbb{P}}}_1), \quad (30)$$

⁶ Considering that the Higgs boson is unique prevents the occurrence of a hierarchy problem [14]

⁷ The orthogonality of \mathcal{R} preserves the property stated in subsection 2.2.2 that the quadratic combination (14) is diagonal for both electroweak and flavour eigenstates; suppose then that there is a single electroweak mass scale M except the vanishing one generated by the breaking of the electroweak symmetry; since, would there be no goldstone, all flavour eigenstates would have the same mass M , the $(mass)^2$ of the flavour components of the charged goldstone G^+ are

$$\begin{aligned} M_{\pi^\pm}^2 &= \frac{M^2}{2} (2 - (G_{ud}^+)^2), \\ M_{K^\pm}^2 &= \frac{M^2}{2} (2 - (G_{us}^+)^2), \\ M_{D^\pm}^2 &= \frac{M^2}{2} (2 - (G_{cd}^+)^2), \\ M_{D_s^\pm}^2 &= \frac{M^2}{2} (2 - (G_{cs}^+)^2). \end{aligned} \quad (27)$$

Our statement that strong interactions determine asymptotic states for mesons can thus participate to creating a mass hierarchy for the latter as a consequence of the breaking of the electroweak symmetry. Of course, electroweak interactions among asymptotic states can occur, in particular through the non-diagonal mass terms which do not cancel any longer after the symmetry is broken.

⁸This makes in particular our results independent of θ_2 , in relation with the fact that one relative phase between the quadruplets has no physical significance.

leading to

$$G_{ud}^+ = c_\theta(c_1 - s_1 s_3) + s_\theta s_1 s_3, \quad (31)$$

$$G_{us}^+ = s_\theta(c_1 - s_1 s_3) - c_\theta s_1 s_3, \quad (32)$$

$$G_{cd}^+ = -s_\theta(c_1 + s_1 s_3) - c_\theta s_1 s_3, \quad (33)$$

$$G_{cs}^+ = c_\theta(c_1 + s_1 s_3) - s_\theta s_1 s_3, \quad (34)$$

where s_θ and c_θ stand for the sine and cosine of the Cabibbo angle.

3.2 Leptonic decays and the hierarchy of decay constants

I show here how the orientation of the Higgs boson in flavour space determines the hierarchy of the pseudoscalar leptonic decay constants.

The leptonic decays are described by the diagram of Fig. 1; because the non-diagonal W – meson coupling only occurs for the three goldstones \vec{G} , their flavour components are the only pseudoscalar mesons which can decay leptonicly. These decays determine the scaling factor \mathbf{b} (21,24) of the flavour eigenstates.

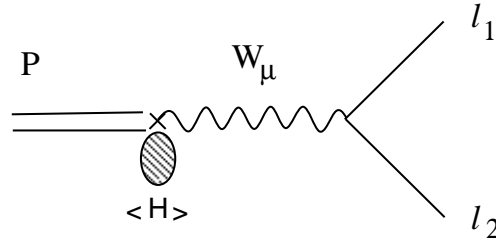


Fig. 1: The leptonic decay of a pseudoscalar meson.

This section gives us the opportunity to explicitly compute a decay amplitude involving asymptotic fields (pseudoscalar mesons) with a normalization different from “1”, while the leptons and gauge fields are normalized to “1”⁹.

The calculation proceeds by using a W propagator in the unitary gauge, and we do it here in analogy with the standard “PCAC” computation.

We have to evaluate

$$_{out}\langle e^+ \nu_e | \pi^+ \rangle_{in} = \langle e^+ \nu_e | \frac{G_F}{\sqrt{2}} L_\mu H^\mu | \pi^+ \rangle, \quad (35)$$

where L_μ and H_μ are respectively the weak leptonic and hadronic currents. H_μ is deduced from the part of the Lagrangian corresponding to the (H, \vec{G}) quadruplet

$$\mathcal{L} \ni -\frac{gv}{2\sqrt{2}} (\partial^\mu G^+ W_\mu^- + \partial^\mu G^- W_\mu^+) + \dots \quad (36)$$

such that, by inserting the vacuum $|0\rangle\langle 0|$ in (35) one gets¹⁰

$$_{out}\langle e^+ \nu_e | \pi^+ \rangle_{in} = \frac{G_F}{\sqrt{2}} \langle e^+ \nu_e | L_\mu | 0 \rangle \langle 0 | v \partial^\mu G^- | \pi^+ \rangle. \quad (37)$$

⁹While the \mathbf{b} factor can be reabsorbed by a simple rescaling of the mesonic fields when the Lagrangian involves only mesons, like for example in the non-linear σ -model, this is more problematic when several types of fields are involved which interact between each other and the different kinetic terms of which come out with different normalizations.

¹⁰This yields the exact result as if computed directly from the diagram of Fig. 1.

In analogy with (25), one has (still in the case of two generations)

$$G^+ = \mathbf{b} (G_{ud}^+ \pi^+ + G_{us}^+ K^+ + G_{cd}^+ D^+ + G_{cs}^+ D_s^+), \quad (38)$$

and, from the relation

$$\pi^+ = \frac{1}{2\mathbf{b}} (c_\theta(P^+(\mathbb{D}_1) + P^+(\mathbb{D}_2)) - s_\theta(P^+(\mathbb{D}_3) + P^+(\mathbb{D}_4))) \quad (39)$$

the inversion of the rotation (28) defining the (H, \vec{G}) multiplet (30) yields

$$\pi^+ = \frac{1}{2\mathbf{b}} ((c_\theta(c_1 - s_1 c_3) + s_\theta s_1 s_3) G^+ + \dots), \quad (40)$$

and thus

$${}_{out}\langle e^+ \nu_e | \pi^+ \rangle_{in} = \frac{1}{2\mathbf{b}} (c_\theta(c_1 - s_1 c_3) + s_\theta s_1 s_3) \frac{G_F}{\sqrt{2}} \langle e^+ \nu_e | L_\mu | 0 \rangle \langle 0 | v \partial^\mu G^- | G^+ \rangle; \quad (41)$$

as the goldstones have been normalized to “1”

$$\langle 0 | G^- | G^+ \rangle = 1 \quad (42)$$

one obtains

$${}_{out}\langle e^+ \nu_e | \pi^+ \rangle_{in} = i v k^\mu \frac{1}{2\mathbf{b}} (c_\theta(c_1 - s_1 c_3) + s_\theta s_1 s_3) \frac{G_F}{\sqrt{2}} \langle e^+ \nu_e | L_\mu | 0 \rangle, \quad (43)$$

where k^μ is the momentum of the incoming pion.

When \mathbf{b} is given by (24), one recovers the standard PCAC result for

$$f_\pi = \left((c_1 - s_1 c_3) + \frac{s_\theta}{c_\theta} s_1 s_3 \right) f_0 = \frac{G_{ud}^+}{c_\theta} f_0. \quad (44)$$

It is easy to find along the same way

$$\begin{aligned} f_K &= \left((c_1 - s_1 c_3) - \frac{c_\theta}{s_\theta} s_1 s_3 \right) f_0 = \frac{G_{us}^+}{s_\theta} f_0, \\ f_D &= \left((c_1 + s_1 c_3) + \frac{c_\theta}{s_\theta} s_1 s_3 \right) f_0 = \frac{G_{cd}^+}{(-s_\theta)} f_0, \\ f_{D_s} &= \left((c_1 + s_1 c_3) - \frac{s_\theta}{c_\theta} s_1 s_3 \right) f_0 = \frac{G_{cs}^+}{c_\theta} f_0. \end{aligned} \quad (45)$$

In the limit $\theta_1 = \theta_3 = 0$ all f 's become identical.

Note the computation of the decay rate does not introduce extra \mathbf{b} factors in the phase space integral because the outgoing states are leptons, which are normalized to “1”.

3.3 The hierarchy of quark condensates

I show now in a precise example how the orientation of the Higgs in flavour space also determines the hierarchy of quark condensates.

By our choice of a unique Higgs boson identified with the scalar entry of $\tilde{\varphi}_1$, we have imposed that it is the only scalar with a non-vanishing vacuum expectation value (VEV). This means a departure from the symmetric case where all diagonal quark condensates have the same VEV and where all non diagonal condensates vanish.

The system of equations to be satisfied by the different VEV's is now:

- for the scalar singlets of the $(\mathbb{S}, \vec{\mathbb{P}})$ multiplets:

$$\langle \tilde{\mathbb{S}}_2 \rangle = \langle \tilde{\mathbb{S}}_3 \rangle = \langle \mathbb{S}^0(\mathbb{D}_4) \rangle = 0; \quad (46)$$

- for the neutral scalars of the $(\mathbb{P}^0, \vec{\mathbb{S}})$ multiplets:

$$\langle \mathbb{S}^3(\mathbb{D}_1) \rangle = \langle \mathbb{S}^3(\mathbb{D}_2) \rangle = \langle \mathbb{S}^3(\mathbb{D}_3) \rangle = \langle \mathbb{S}^3(\mathbb{D}_4) \rangle = 0, \quad (47)$$

where the notation $\langle \mathbb{S} \rangle$ used above is a shortcut for $\langle \bar{\Psi} \mathbb{S} \Psi \rangle$.

One case for which these equations can be solved approximately is for example $s_3 \approx 0, c_3 \approx 1$; one finds

$$\begin{aligned} \langle \bar{c}c \rangle &\approx \frac{c_1 + s_1}{c_1 - s_1} \langle \bar{u}u \rangle, \\ \langle \bar{d}d \rangle &\approx \frac{c_1 - s_1(c_\theta^2 - s_\theta^2)}{c_1 - s_1} \langle \bar{u}u \rangle, \\ \langle \bar{s}s \rangle &\approx \frac{c_1 + s_1(c_\theta^2 - s_\theta^2)}{c_1 - s_1} \langle \bar{u}u \rangle, \\ \langle \bar{d}s \rangle &= \langle \bar{s}d \rangle \approx -2 \frac{s_1 s_\theta c_\theta}{c_1 - s_1} \langle \bar{u}u \rangle, \\ \langle \bar{u}c \rangle &= \langle \bar{c}u \rangle = 0. \end{aligned} \quad (48)$$

For $c_1 > 0$, $s_1 < 0$, one gets accordingly the hierarchy

$$\langle \bar{u}u \rangle > \langle \bar{d}d \rangle > \langle \bar{s}s \rangle > \langle \bar{c}c \rangle > \langle \bar{d}s \rangle = \langle \bar{s}d \rangle > \langle \bar{u}c \rangle = \langle \bar{c}u \rangle = 0; \quad (49)$$

it agrees with the one generally admitted from the Gell-Mann-Oakes-Renner relation; the non-vanishing of non-diagonal quark condensates, which appears in a natural manner here, has been debated in the past when dealing with kaon decays [15].

3.4 The coupling of the Higgs boson to pseudoscalar mesons

I determine here the coupling of the Higgs boson to pseudoscalar mesons in terms of leptonic decay constants and CKM mixing angles.

The “mexican hat” potential introduced for $\tilde{\varphi}_1 = (H, \vec{G})$

$$V(H, \vec{G}) = -\frac{\sigma^2}{2} \tilde{\varphi}_1 \otimes \tilde{\varphi}_1 + \frac{\lambda}{4} (\tilde{\varphi}_1 \otimes \tilde{\varphi}_1)^{\otimes 2} \quad (50)$$

which triggers the breaking of the electroweak symmetry yields in particular a coupling between the Higgs and the goldstones

$$\mathcal{L} \ni -\frac{\lambda}{\sqrt{2}} v h (2G^+ G^- + G^3 G^3). \quad (51)$$

Let a, b the flavour indices of a pseudoscalar meson \mathcal{P}_{ab} , with mass M_{ab} transforming like the diquark operator $\bar{q}_a \gamma_5 q_b$ (for example $\mathcal{P}_{ud}^+ = \pi^+$); the goldstone G^+ writes, according to (11,12,21) and in analogy with (38)

$$G^+ = \sum_{a,b} G_{ab}^+ \Pi_{ab}^+ = \mathbf{b} \sum_{a,b} G_{ab}^+ \mathcal{P}_{ab}^+. \quad (52)$$

Making use of the equivalent of relations (44,45) in the case of three generations, one gets in particular for two charged outgoing flavour eigenstates Π_{ab}^+ and Π_{cd}^- the coupling ¹¹

$$-i \sqrt{2} \lambda v \sum_{a,b} \sum_{c,d} \frac{f_{ab} f_{cd}}{f_0^2} V_{ab} V_{cd}^\dagger h \Pi_{ab}^+ \Pi_{cd}^- \quad (53)$$

where f_{ab} is the leptonic decay constant of \mathcal{P}_{ab} and V_{ab} the corresponding entry of the CKM mixing matrix.

A dominant feature in (53) is the presence of the CKM mixing angles V_{ab} , which, unlike what happens at the quark level for Yukawa couplings, can strongly damp the corresponding coupling independently of the mass of the outgoing particles. This is specially the case for B^\pm mesons, since the corresponding V_{ub} lies far from the diagonal in the CKM mixing matrix.

I now use the above results to investigate decays of the Z boson which are mediated by the Higgs.

¹¹The Lagrangian is always written with fields which are normalized to “1”, and the corresponding couplings are used to compute S -matrix elements between states which are also normalized to “1”. The physical S -matrix elements involving asymptotic mesons that we need to compute are then deduced by introducing the appropriate \mathbf{b} factors.

4 Some decays $Z \rightarrow e^+ e^- \mathcal{P}_i^+ \mathcal{P}_j^-$

I shall always deal with the cases when the outgoing leptons are electrons; when they are muons, the results are very much alike, because the relative difference in the available phase space is very small.

4.1 The decay $Z \rightarrow e^+ e^- B^+ B^-$

There are two types of contributions shown in Figs. 2a,2b.

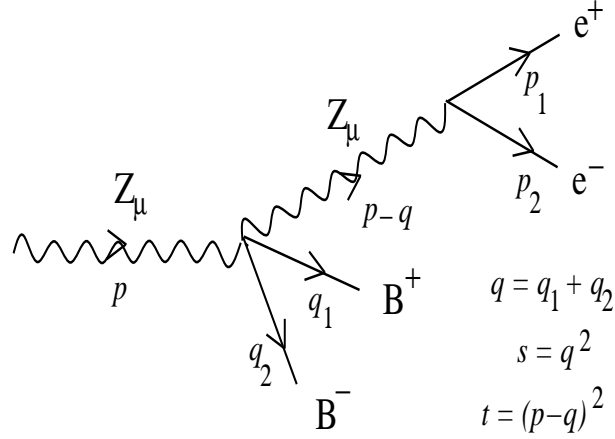


Fig. 2a: The decay $Z \rightarrow e^+ e^- B^+ B^-$: direct coupling.

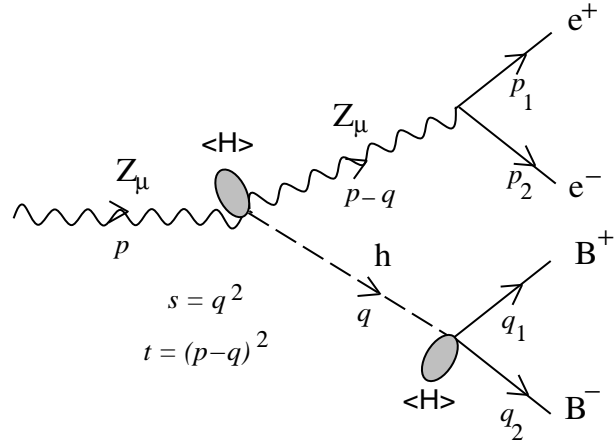


Fig. 2b: The decay $Z \rightarrow e^+ e^- B^+ B^-$: the contribution of the Higgs boson.

That there exists a direct coupling of two gauge bosons to two mesons differs from the standard model with quarks, where the mesons can only originate from the “hadronization” of the two quarks coupled to the Higgs boson through a Yukawa coupling.

We need to compute the amplitude $\mathcal{A}_{Z \rightarrow B^+ B^- e^+ e^-}$

$$\mathcal{A}_{Z \rightarrow B^+ B^- e^+ e^-} = \text{out} \langle e^+ e^- B^+ B^- | Z_\mu \rangle_{\text{in}} \quad (54)$$

where the asymptotic fields Z_μ, e^\pm are normalized to “1” while the B mesons are not (see subsection 2.2.1). This introduces \mathfrak{b} factors according to

$$\mathcal{A}_{Z \rightarrow B^+ B^- e^+ e^-} = \frac{1}{\mathfrak{b}^2} \text{out} \langle e^+ e^- \Pi_{ub}^+ \Pi_{bu}^- | Z_\mu \rangle_{\text{in}} \quad (55)$$

where we have introduced the flavour eigenstates Π_{ab} normalized to “1” which are also used to express the Lagrangian. Eq. (55) can be calculated with standard rules.

The total coupling of two Z bosons to $\Pi_{ub}^+ \Pi_{bu}^-$ reads (including the “ i ” coming from $\exp(iS)$)

$$i \frac{g^2}{2c_W^2} \left(1 + \frac{1}{2} \left(\frac{f_B}{f_0} \right)^2 V_{ub} V_{bu}^\dagger \frac{M_H^2}{q^2 - M_H^2} \right) \quad (56)$$

where the first contribution comes from the direct coupling and the second from the one involving the Higgs boson; c_W is the cosine of the Weinberg angle; q is the Higgs momentum, the mass of which is (see (50,9))

$$M_H^2 = \lambda v^2. \quad (57)$$

Note that the mixing angles $V_{ub} V_{bu}^\dagger$ only appear in the Higgs contribution.

A second potential source of damping appears in (56) since there can be a destructive interference between the direct contribution and the one with the Higgs boson.

The decay rate is expressed as a double integral over the square of the Higgs momentum $s = q^2$ and that of the virtual Z momentum $t = (p - q)^2$:

$$\begin{aligned} \Gamma_{Z \rightarrow e^+ e^- B^+ B^-} &= \frac{1}{512 \sqrt{2} \pi^5} (1 - 2s_W^2) M_Z^3 G_F^3 \\ &\int_{4M_B^2}^{(M_Z - 2m_e)^2} ds \frac{\lambda^{1/2}(s, M_B^2, M_B^2)}{s} \left(1 + \frac{1}{2} \left(\frac{f_B}{f_0} \right)^2 V_{ub} V_{bu}^\dagger \frac{M_H^2}{s - M_H^2} \right)^2 \\ &\int_{4m_e^2}^{(M_Z - \sqrt{s})^2} dt \frac{\lambda^{1/2}(M_Z^2, s, t)}{(t - M_Z^2)^2} \left(t + \frac{\lambda(M_Z^2, s, t)}{12 M_Z^2} \right), \end{aligned} \quad (58)$$

where M_Z is the mass of the Z gauge boson,¹² M_B the mass of the B^\pm pseudoscalar mesons, m_e the mass of the electron¹³ and s_W the sine of the Weinberg angle; $\lambda(u, v, w)$ is the fully symmetric function

$$\lambda(u, v, w) = u^2 + v^2 + w^2 - 2uv - 2uw - 2vw; \quad (59)$$

All \mathbf{b} factors cancel in the decay rate: the $1/\mathbf{b}^4$ coming from the amplitude (55) squared exactly matches the two factors \mathbf{b}^2 coming from the phase-space measures (23) for the two outgoing mesons.

The kinematical intervals being respectively $s \in [4M_B^2, (M_Z - 2m_e)^2]$ and $t \in [4m_e^2, (M_Z - \sqrt{s})^2]$, for $M_H^2 < (M_Z - 2m_e)^2$, the potential divergence of the decay rate due to the (squared) propagator of the Higgs has to be smoothed out by introducing a width for the latter; we restrict ourselves to the two-mesons channels, as depicted in Fig. 3.

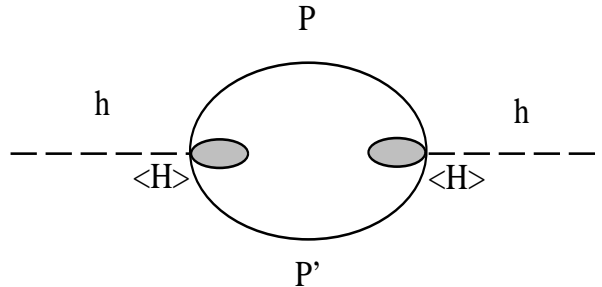


Fig. 3: Introducing a width for the Higgs boson.

The couplings of the Higgs to two charged mesons are the same as the ones used above and studied in subsection 3.4; we shall however neglect in the computation of its width the misalignment of the Higgs with respect to

¹²With the normalizations chosen here one has $M_W^2 = g^2 v^2 / 8$ and $G_F / \sqrt{2} = g^2 / 8 M_W^2 = 1/v^2$, where g is the $SU(2)_L$ coupling constant.

¹³The mass of the electron has been neglected in the computation of the square of the amplitude.

$\mathbb{S}^0(\mathbb{D}_1)$, consider all ratios $f_{ab}/f_0 \approx 1$ and only take into account the channels which are not damped by small mixing angles (combinations of π^\pm and D_s^\pm in the charged sector). The neutral mesons ($\pi^0, \eta, \eta_c, \eta_b$) are incorporated along the same lines.

We thus replace in (58)

$$\begin{aligned} & \left(1 + \frac{1}{2} \left(\frac{f_B}{f_0} \right)^2 V_{ub} V_{bu}^\dagger \frac{M_H^2}{(s - M_H^2)} \right)^2 \longrightarrow \\ & = 1 + \frac{\left(\frac{f_B}{f_0} \right)^2 V_{ub} V_{bu}^\dagger M_H^2 (s - M_H^2) + \left(\frac{1}{2} \left(\frac{f_B}{f_0} \right)^2 V_{ub} V_{bu}^\dagger \right)^2 M_H^4}{(s - M_H^2)^2 + \left(\frac{G_F M_H^4}{32\pi\sqrt{2}} \right)^2 \left(\sum_{ab} \sum_{cd} V_{ab} V_{ba}^\dagger V_{cd} V_{dc}^\dagger \frac{\lambda^{1/2}(s, M_{ab}^2, M_{cd}^2)}{s} \right)^2} \end{aligned} \quad (60)$$

where the $\sum_{ab} \sum_{cd}$ in (60) are performed over all above mentioned couples of $J = 0$ pseudoscalar mesons with flavour indices (ab) and (cd) and masses M_{ab} and M_{cd} ; in the charged sector, the V_{ab} 's are restricted to the diagonal entries of the CKM mixing matrix, and stand for 1 in the neutral sector (see (5) with $\mathbb{D} = 1$).

To maximize the decay rate, we take $f_B/f_0 \approx 10$ and V_{ub} is chosen at the upper limit of the experimental bounds [16]

$$V_{ub} \approx 4 \cdot 10^{-3}. \quad (61)$$

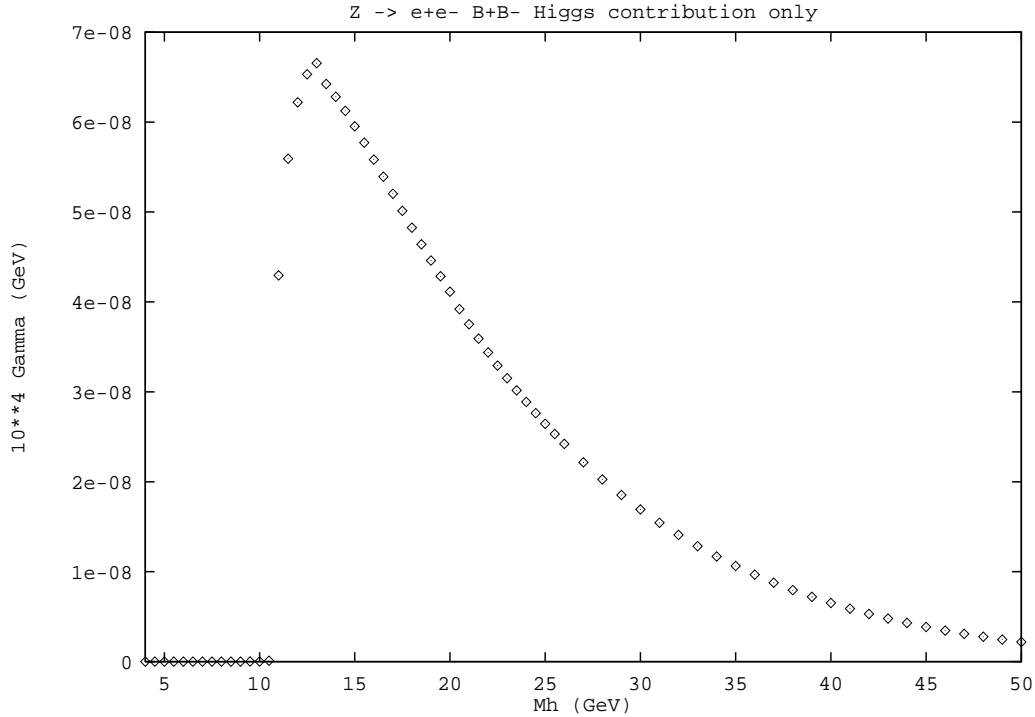


Fig. 4: $10^4 \times$ the Higgs contribution to the decay rate $Z \rightarrow e^+ e^- B^+ B^-$.

The Higgs contribution, shown in Fig. 4, exhibits a resonance-like behaviour, with a maximum of $\approx 6.6 \cdot 10^{-12} \text{ GeV}$ for $M_H \approx 13 \text{ GeV}$, which is extremely small¹⁴. The “background” coming from the contribution where no Higgs is involved is nearly 1000 times larger than the Higgs contribution at its maximum and is itself outside

¹⁴The numerical integrations have been performed by the method of Newton and dividing, for each point of the graphs, the two dimensional domain of integration into $5 \cdot 10^3 \times 10^3$ cells, ensuring a perfect stability.

the reach of present observations, with a rate ¹⁵

$$\Gamma_{Z \rightarrow e^+ e^- B^+ B^-}^{no\ Higgs} = 5 \cdot 10^{-9} \text{ GeV}, \quad (62)$$

to be compared with the total width of the Z boson $\Gamma_Z \approx 2.4 \text{ GeV}$; suppose that one can analyze $20 \cdot 10^6$ Z decays [17]; ten identified decays into two leptons and two pseudoscalar mesons would correspond to a fraction $5 \cdot 10^{-7}$ of all and to a partial width $\Gamma_{part} = 1.2 \cdot 10^{-6} \text{ GeV}$; it seems consequently reasonable to set an (optimistic) threshold of observability above a partial width

$$\Gamma_{obs} \geq 10^{-6} \text{ GeV}. \quad (63)$$

Accordingly, it appears useless to look for a Higgs boson like the one described above in the decay $Z \rightarrow e^+ e^- B^+ B^-$.

4.2 The decay $Z \rightarrow e^+ e^- D_s^+ D_s^-$

I then analyze the decay $Z \rightarrow e^+ e^- D_s^+ D_s^-$; the corresponding rate is computed from formulae similar to (58,60), with the appropriate substitutions concerning the masses, leptonic decay constants and mixing angles. The damping due to CKM mixing angles is now negligible, which makes the Higgs contribution dominate over the direct coupling of the Z to two mesons. It appears consequently as a better reaction to look for the Higgs boson.

The background coming from the direct coupling of the Z to two mesons is negligible:

$$\Gamma_{Z \rightarrow e^+ e^- D_s^+ D_s^-}^{no\ Higgs} = 7.6 \cdot 10^{-9} \text{ GeV}. \quad (64)$$

The results are shown in Figs. 5.

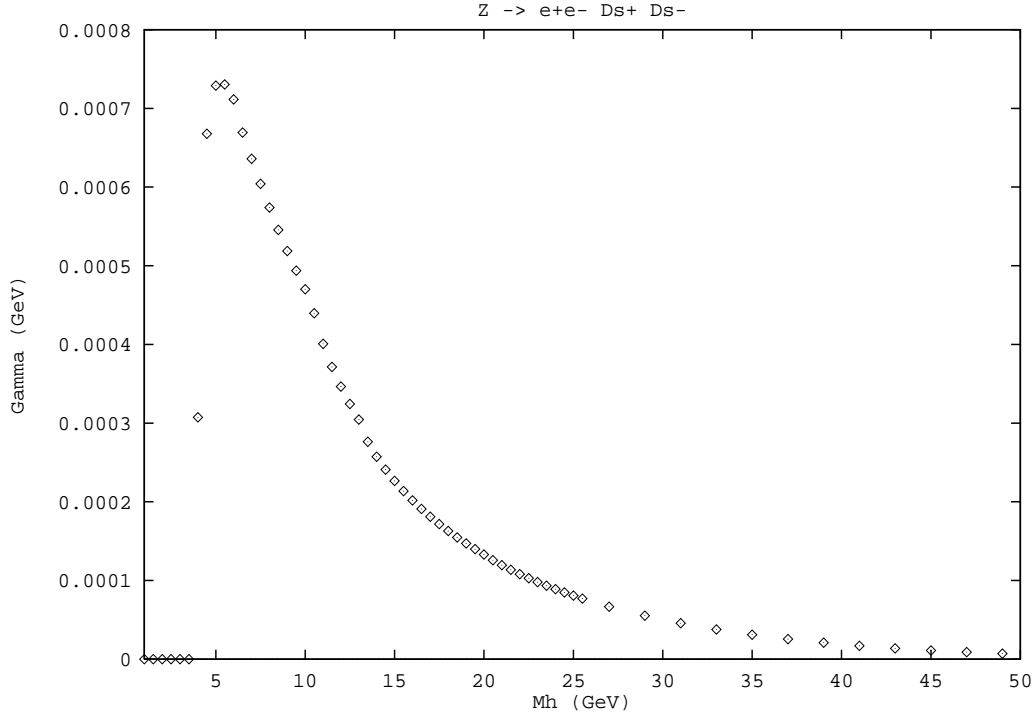


Fig. 5a: The decay rate $Z \rightarrow e^+ e^- D_s^+ D_s^-$.

¹⁵The destructive interference effects between the two contributions is of course negligible.

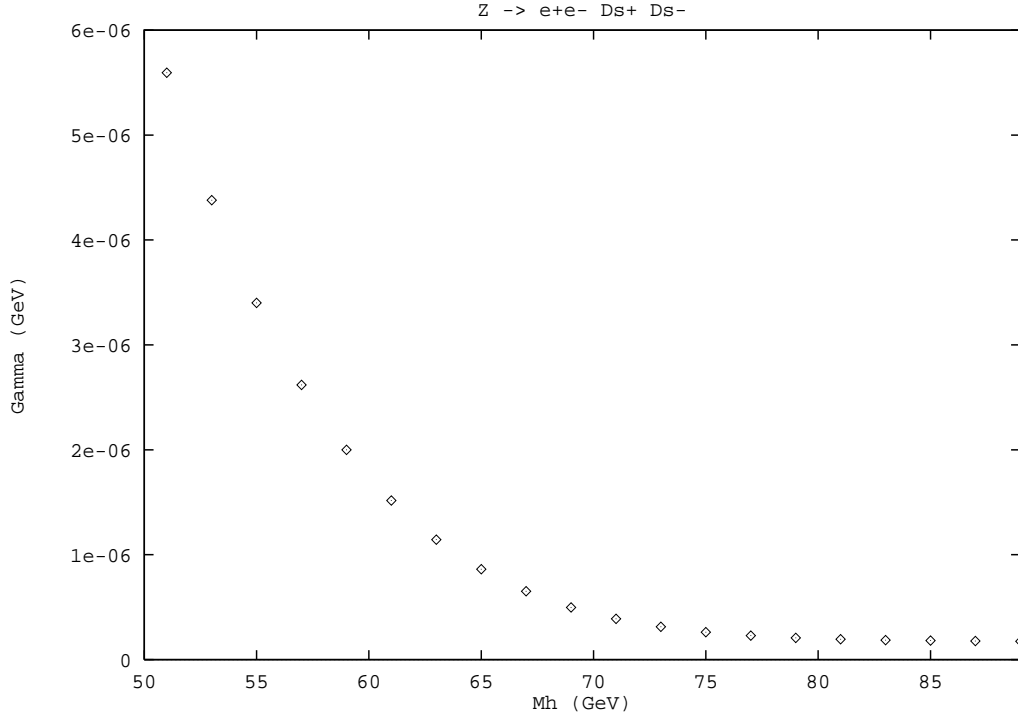


Fig. 5b: The decay rate $Z \rightarrow e^+e^- D_s^+ D_s^-$.

The threshold of observability (63) is satisfied for $M_H \leq 65 \text{ GeV}$.

This is to be compared with the present lower bound of the four LEP experiments for the Higgs boson of the standard model $M_{H \text{ standard}}^{LEP} \geq 90 \text{ GeV}$ at 95% confidence level [18]. In this mass range, there is no hope to detect the Higgs that is proposed here, but it could have been missed at lower masses (but high enough for the missing energy channel to be undetectable [1]) because of a too low statistics.

4.3 The decay $Z \rightarrow e^+e^- K^\pm \pi^\mp$

There also exist in this approach decays which do not occur in the Glashow-Salam-Weinberg model for quarks and characterize a composite Higgs like has been introduced above; they are the ones corresponding to “flavour changing neutral currents” in the scalar sector. They could furthermore be easily identified experimentally if produced with enough statistics.

From (51) and (38) or its generalization to three generations, it appears that the Higgs can decay into final states like $K^\pm \pi^\mp, D^\pm K^\mp \dots$, and that, due to the choice of the kinetic terms as stated in subsections 2.2.3 and 2.2.2 and to the property of diagonalization of the corresponding quadratic invariant (14), the background described in the two previous decays is now absent.

I study here the $e^+e^- K^\pm \pi^\mp$ final state, the amplitude of which benefits from a moderate damping by the mixing angles, only $s_\theta c_\theta$. The decay rate is computed from (58,60) where the factor “1” in each of them, corresponding to the direct coupling, is dropped since the latter does not exist any more.

The results for the decay rate are plotted in Fig. 6.

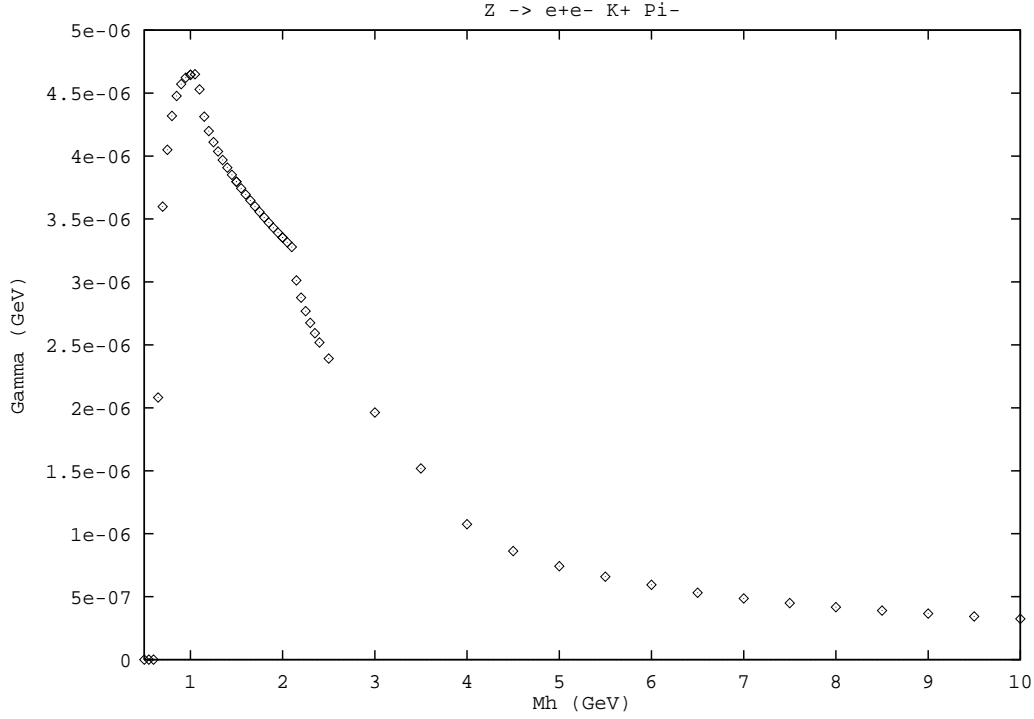


Fig. 6: The decay rate $Z \rightarrow e^+e^- K^\pm \pi^\mp$.

It exhibits the same resonance-like behaviour as the previous decays, but has its maximum $\Gamma_{max} \approx 4.7 \cdot 10^{-6} \text{ GeV}$ at a low Higgs mass $M_H \approx 1 \text{ GeV}$. The cusps that can be seen on the curve have been checked to correspond to the opening of the different two-mesons channels in the propagator of the Higgs boson.

The resonance is rather sharp, and it appears that for $M_H > 7 \text{ GeV}$ the width of the process is lower than the threshold (63) above which there is no hope that it be observed.

As a Higgs mass $M_H < 7 \text{ GeV}$ would have been detected by the corresponding missing energy [1] (with the possible *caveat* of footnote 2), there seems unfortunately to be no hope to observe the characteristic decays mentioned above in a foreseeable future.

5 Conclusion

Answering a demand that alternatives to the strictly standard Higgs boson or to its supersymmetric extensions be searched for. I have proposed one, which, though it shares many similarities with the standard model, exhibits a Higgs boson that is still more elusive; it may not however be beyond experimental reach.

It has been incorporated in a framework where the fields in the Lagrangian are tightly related with the $J = 0$ mesonic states observed asymptotically; I have related the orientation of the Higgs in flavour space to leptonic decay constants of pseudoscalar mesons and to the CKM mixing angles, such that the quartic potential which triggers the breaking of the electroweak symmetry has been expressed in terms of these parameters and of the mass of the Higgs itself. It includes in particular the coupling of the Higgs to two pseudoscalar mesons, which is no longer triggered by Yukawa couplings to quarks followed by an hadronization process as it used to be in the standard model. New links have thus been provided, which enabled the study of the disintegration of the Z gauge boson into two leptons and two pseudoscalar mesons.

I showed that this Higgs boson has interesting and specific properties, in particular that it can trigger flavour changing neutral currents, by decaying into final states of the type $K^\pm \pi^\mp$; unfortunately, detecting those decays would require a tremendous increase of the available number of Z bosons.

The decay $Z \rightarrow e^+e^-D_s^+D_s^-$ is the best candidate because it is not damped by small mixing angles and the background due to the direct coupling of the Z to two mesons is negligible. A Higgs with mass lower than 65 GeV could have been missed.

Unlike in the standard model for quarks, the final state $e^+e^-B^+B^-$ appears to be undetectable because of the presence of the CKM mixing angles in the Higgs coupling to two pseudoscalar mesons; the Higgs contribution is furthermore screened by a background which, though much larger, is itself undetectable.

I did not explicitly present here the results for another channel which is not suppressed by small mixing angles, $Z \rightarrow e^+e^-\pi^+\pi^-$: the reason is that the corresponding decay rate is then peaked at very low Higgs masses $M_H < 1\text{ GeV}$, for which such a particle could not have escaped detection in the missing energy channel; for higher masses it becomes again absolutely undetectable.

For large Higgs masses ($M_H > M_W$), the type of decays studied here is undetectable, and more standard considerations have to be pursued.

Acknowledgments: *it is a pleasure to thank F. Boudjema for critics and suggestions.*

List of Figures

Fig. 1: Leptonic decay of a pseudoscalar meson;

Figs. 2: The two contributions to the decay $Z \rightarrow e^+ e^- B^+ B^-$;

Fig. 3: Introducing a width for the Higgs boson;

Fig. 4: $10^4 \times$ the Higgs contribution to the rate of the decay $\Gamma_{Z \rightarrow e^+ e^- B^+ B^-}$ as a function of the Higgs mass;

Figs. 5: The decay rate $\Gamma_{Z \rightarrow e^+ e^- D_s^+ D_s^-}$ as a function of the Higgs mass;

Fig. 6: The decay rate $\Gamma_{Z \rightarrow e^+ e^- \pi^+ K^-}$ as a function of the Higgs mass.

Appendix

A Diagonalizing eq. (14) in the basis of strong eigenstates: a choice of \mathbb{D} matrices

The property stated in subsection 2.2.2 is most simply verified for the “non-rotated” $SU(2)_L \times U(1)$ group and representations which corresponds to setting $\mathbb{K} = \mathbb{I}$ in (2,5).

A.1 $N = 2$ (1 generation).

Trivial case: \mathbb{D} is a number.

A.2 $N = 4$ (2 generations).

The four 2×2 \mathbb{D} matrices (3 symmetric and 1 antisymmetric) can be taken as

$$\mathbb{D}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbb{D}_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \mathbb{D}_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \mathbb{D}_4 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (65)$$

A.3 $N = 6$ (3 generations).

The nine 3×3 \mathbb{D} matrices (6 symmetric and 3 antisymmetric), can be taken as

$$\begin{aligned} \mathbb{D}_1 &= \sqrt{\frac{2}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\ \mathbb{D}_2 &= \frac{2}{\sqrt{3}} \begin{pmatrix} \sin \alpha & 0 & 0 \\ 0 & \sin(\alpha \pm \frac{2\pi}{3}) & 0 \\ 0 & 0 & \sin(\alpha \mp \frac{2\pi}{3}) \end{pmatrix}, \mathbb{D}_3 = \frac{2}{\sqrt{3}} \begin{pmatrix} \cos \alpha & 0 & 0 \\ 0 & \cos(\alpha \pm \frac{2\pi}{3}) & 0 \\ 0 & 0 & \cos(\alpha \mp \frac{2\pi}{3}) \end{pmatrix}, \\ \mathbb{D}_4 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \mathbb{D}_5 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \\ \mathbb{D}_6 &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \mathbb{D}_7 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \mathbb{D}_8 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \mathbb{D}_9 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \end{aligned} \quad (66)$$

where α is an arbitrary phase.

B The normalization of the asymptotic mesons and the Gell-Mann-Oakes-Renner relation

From (12), (21) and (24) one gets for the pseudoscalar asymptotic meson \mathcal{P}_{ab} associated with the matrix \mathbb{M}_{ab} (see subsection 2.2.1)

$$\mathcal{P}_{ab}(\mathbb{M}_{ab}) = i \frac{2f_0}{\langle \bar{\Psi} \mathbb{H} \Psi \rangle} \bar{\Psi} \gamma_5 \mathbb{M}_{ab} \Psi, \quad (67)$$

where the matrix \mathbb{H} which describes the Higgs boson H in flavour space has been defined in (10).

To make the link with the customary quark picture, one recalls that, in QCD, the pseudoscalar density $i\bar{\Psi}\gamma_5\mathbb{M}_{ab}\Psi$ is related to the divergence of the axial current with flavour indices a, b

$$A_{ab}^\mu = \bar{\Psi} \gamma^\mu \gamma_5 \mathbb{M}_{ab} \Psi = \bar{q}_a \gamma^\mu \gamma_5 q_b \quad (68)$$

by

$$\partial_\mu A_{ab}^\mu = i(m_a + m_b) \bar{\Psi} \gamma_5 \mathbb{M}_{ab} \Psi, \quad (69)$$

where m_a and m_b are the “masses” of the quarks q_a and q_b .

Now, one form of the PCAC statement is that $\partial_\mu A_{ab}^\mu$ is proportional to the interpolating field of the observed pseudoscalar meson \mathcal{P}_{ab} according to

$$\partial_\mu A_{ab}^\mu = f_{ab} M_{ab}^2 \mathcal{P}_{ab}, \quad (70)$$

where M_{ab} is the mass of the meson. This transforms (67) into

$$\mathcal{P}_{ab}(\mathbb{M}_{ab}) = \frac{2f_0}{\langle \bar{\Psi} \mathbb{H} \Psi \rangle} \frac{f_{ab} M_{ab}^2}{m_a + m_b} \mathcal{P}_{ab} \quad (71)$$

which entails

$$(m_a + m_b) \langle \bar{\Psi} \mathbb{H} \Psi \rangle = 2 f_0 f_{ab} M_{ab}^2. \quad (72)$$

Eq. (72) is, up to a sign (unimportant since it can be introduced in (67)), the equivalent of the Gell-Mann-Oakes-Renner relation [9], showing the consistency of our approach with a more traditional one.

References

- [1] *see for example:*
J.F. GUNION, H.E. HABER, G. KANE & S. DAWSON: “*The Higgs Hunter’s Guide*”, *Frontiers in Physics* (Addison-Wesley 1990);
M. DITTMAR: “*Searching for the Higgs and other Exotic Objects*”, lectures given at the 30-th *École d’Été de Physique des Particules*, Marseille 7-11 Septembre 1998;
and references therein.
- [2] P.W. HIGGS: “*Broken symmetries, massless particles and gauge fields*”, *Phys. Lett.* 12 (1964) 132;
P.W. HIGGS: “*Broken symmetries and the masses of gauge bosons*”, *Phys. Rev. Lett.* 13 (1964) 508;
P.W. HIGGS: “*Spontaneous symmetry breakdown without massless bosons*”, *Phys. Rev.* 145 (1966) 1156;
F. ENGLERT & R. BROUT: “*Broken symmetry and the mass of gauge vector mesons*”, *Phys. Rev. Lett.* 13 (1964) 321;
G.S. GURALNIK, C.R. HAGEN & T.W.B. KIBBLE: “*Global conserved laws and massless particles*” *Phys. Rev. Lett.* 13 (1964) 585;
T.W.B. KIBBLE: “*Symmetry Breaking in Non-Abelian Gauge Theories*”, *Phys. Rev.* 155 (1967) 1554.
- [3] S. L. GLASHOW: “*Partial-symmetry of weak interactions*”, *Nucl. Phys.* 22 (1961) 579;
A. SALAM: “*Weak and electromagnetic interactions*”, in “*Elementary Particle Theory: Relativistic Groups and Analyticity*” (Nobel symposium No 8), edited by N. Svartholm (Almqvist and Wiksell, Stockholm 1968);
S. WEINBERG: “*A model of leptons*”, *Phys. Rev. Lett.* 19 (1967) 1264.
- [4] *see for example:*
M. SPIRA & P.M. ZERWAS: “*Electroweak Symmetry Breaking and Higgs Physics*”, *Lectures at 36th Internationale Universitätswochen für Kern-und Teilchenphysik, Shladming 1997*, preprint DESY 97-261, CERN-TH/97-379, hep-ph/9803257, and references therein.
- [5] *see for example:*
R.S. CHIVUKULA: “*Models of Electroweak Symmetry Breaking*”, lectures at *Les Houches Summer School* (1997), hep-ph/9803219, and references therein.
- [6] B. MACHET: “*Chiral Scalar Fields, Custodial Symmetry in Electroweak $SU(2)_L \times U(1)$, and the Quantization of the Electric Charge*”, hep-ph/9606239, *Phys. Lett. B* 385 (1996) 198-208.
- [7] *see for example:*
W. MARCIANO & H. PAGELS: “*Quantum Chromodynamics*”, *Physics Reports C* 36 (1978) 137-276;
C. QUIGG: “*Gauge Theories of the Strong, Weak and Electromagnetic Interactions*”, *Frontiers in Physics* (Addison Wesley, 1994);
and references therein.
- [8] N. CABIBBO: “*Unitary symmetry and leptonic decays*”, *Phys. Lett.* 10 (1963) 531;
M. KOBAYASHI & T. MASKAWA: “*CP-Violation in the Renormalizable Theory of Weak Interactions*”, *Prog. Theor. Phys.* 49 (1973) 652.
- [9] M. GELL-MANN, R.J. OAKES & B. RENNER: “*Behaviour of Current Divergences under $SU(3) \times SU(3)$* ”, *Phys. Rev.* 175 (1968) 2195.
- [10] M. GELL-MANN: “*A schematic model of baryons and mesons*”, *Phys. Lett.* 8 (1964) 214.
- [11] M. GELL-MANN: “*Isotopic Spin and New Unstable Particles*”, *Phys. Rev.* 92 (1953) 833;
K. NISHIJIMA: “*Charge Independence Theory of V particles*”, *Prog. Theor. Phys.* 13 (1955) 285.
- [12] M. BELLON & B. MACHET: “*The Standard Model of Leptons as a Purely Vectorial Theory*”, *Phys. Lett. B* 313 (1993) 141-148, hep-ph/9305212;
B. MACHET: “*Leptonic Custodial Symmetry, Quantization of the Electric Charge, and the Neutrino in the Standard Model*”, *Mod. Phys. Lett. A* 11 (1996) 2297-2307, hep-ph/9606308.

- [13] B. MACHET: “Some aspects of pion physics in a dynamically broken abelian gauge theory”, *hep-ph/9306285*, *Mod. Phys. Lett. A* 9 (1994) 3053-3062; “Comments on the Standard Model of electroweak interactions”, *hep-ph/9406349*, *Int. J. Mod. Phys. A* 11 (1996) 29-63; “A new mechanism for CP violation and pseudoscalar mesons revisited”, preprint PAR-LPTHE 96/45, *hep-ph/9611262*, accepted for publication in *Phys. Lett. B*; “An electroweak $SU(2)_L \times U(1)$ gauge theory of $J = 0$ mesons, extended version of the talk “Custodial symmetry in a $SU(2)_L \times U(1)$ gauge theory of $J = 0$ mesons” given at the 2nd International Symposium on Symmetries in Subatomic Physics, Seattle (Washington), June 25th-29th 1997, preprint PAR-LPTHE 97/32, *hep-ph/9709278*; “Indirect CP violation in an electroweak $SU(2)_L \times U(1)$ gauge theory of chiral mesons”, preprint PAR-LPTHE 98/12, *hep-ph/9804417*; “Large breaking of isospin symmetry in an electroweak $SU(2)_L \times U(1)$ gauge theory of $J = 0$ mesons: the case of $K \rightarrow \pi\pi$ decays”, preprint PAR-LPTHE 98/21, *hep-ph/9805458*.
- [14] E. GILDENER & S. WEINBERG: “Symmetry breaking and scalar bosons”, *Phys. Rev. D* 13 (1976) 3333; E. GILDENER: “Gauge-symmetry hierarchies”, *Phys. Rev. D* 14 (1976) 1667.
- [15] see for example:
M. LEURER: “The tadpole in the chiral Lagrangian of K decays”, *Phys. Lett. B* 201 (1988) 128.
- [16] “THE REVIEW OF PARTICLE PROPERTIES”, *The European Physical Journal C* 3 (1998).
- [17] see for example:
THE LEP COLLABORATIONS, THE LEP ELECTROWEAK WORKING GROUP AND THE SLD HEAVY FLAVOUR AND ELECTROWEAK GROUPS: “A Combination of Preliminary Electroweak Measurements and Constraints on the Standard Model”, preprint CERN-EP/99-15 (Feb. 1999).
- [18] see for example:
T. GREENING: “Search for the Standard Model Higgs Boson at LEP”, *hep-ex/9903013*.